

Factoring Polynomials

- any # terms 1. Common Factoring ← * Always try this first!
- 4 terms 2. Grouping Factoring
- 3 terms 3. Trinomial Factoring "a=1" (Easy)
"a≠1" (Harder) Decomposition Method
- 2 terms 4. Difference of Squares

- Factoring means to write as a product

- Factor 15

$$15 = (3)(5)$$

Simplify ("Multiply it out")

$$2x(5x + 3) = 10x^2 + 6x$$

Factoring is the opposite of this

Factor the following binomials by using common factoring.

1. $4x - 10$

2. $x^5 - 2x^3$

3. $3x^2 - 12x$

4. $5x^3 + 20x^2$

5. $2x^2y - 16xy^2$

6. $-x^3 + x^4y$

1. $4x - 10 = 2(2x - 5)$

2. $x^5 - 2x^3 = x^3(x^2 - 2)$

3. $3x^2 - 12x = 3x(x - 4)$

4. $5x^3 + 20x^2 = 5x^2(x + 4)$

5. $2x^2y - 16xy^2$
 $= 2xy(x - 8y)$

6. $-x^3 + x^4y$
 $= -x^3(1 - xy)$

Factor the following trinomials by using common factoring.

7. $6m^3n^2 - 2m^2n^3 + 4m^2n^2$

8. $9xyz - 3x^2y + 6xyz^2$

9. $16p^2q^2 - 8p^2q^4 - 24p^4q^2$

$$\begin{aligned} 7. \quad & 6m^3n^2 - 2m^2n^3 + 4m^2n^2 \\ & = 2m^2n^2(3m - n + 2) \end{aligned}$$

$$\begin{aligned} 8. \quad & 9xyz - 3x^2y + 6xyz^2 \\ & = 3xy(3z - x + 2z^2) \end{aligned}$$

$$\begin{aligned} 9. \quad & 16p^2q^2 - 8p^2q^4 - 24p^4q^2 \\ & = 8p^2q^2(2 - q^2 - 3p^2) \end{aligned}$$

Factor each expression below by identifying a binomial that is a common factor.

10. $3x(x + y) - 4(x + y)$

11. $2a(a - b) + 5b(a - b)$

12. $3p(p + 5) - 7(p + 5)$

10. $3x(x+y) - 4(x+y) = (x+y)(3x-4)$

11. $2a(a-b) + 5b(a-b) = (a-b)(2a+5b)$

12. $3p(p+5) - 7(p+5) = (p+5)(3p-7)$

Factor each expression by using grouping.

1. $ay + a + 4y + 4$

1. $ay + a + 4y + 4 = a(y+1) + 4(y+1)$
 $= (y+1)(a+4)$

2. $x^3 + 2x^2 + 3x + 6$

2. $x^3 + 2x^2 + 3x + 6 = x^2(x+2) + 3(x+2) = (x+2)(x^2+3)$

3. $x^3 - 7x^2y - 3xy + 21y^2$

3. $x^3 - 7x^2y - 3xy + 21y^2 = x^2(x-7y) - 3y(x-7y)$
 $= (x-7y)(x^2-3y)$

4. $xy + 3xz + 2yz + 6z^2$

4. $xy + 3xz + 2yz + 6z^2 = x(y+3z) + 2z(y+3z)$
 $= (y+3z)(x+2z)$

Factor each expression by using grouping. Remember to look for a common factor first.

5. $3x^2 - 12x - 3xy + 12y$

6. $2x^2y - 2xy - 2xy^2 + 2y^2$

7. $-4x^2 + 8xy - 20x + 40y$

8. $60x^2 - 30xy + 10xy^2 - 5y^3$

5. $3x^2 - 12x - 3xy + 12y$

$$= 3 \left[\underline{x^2 - 4x} - \underline{xy + 4y} \right]$$

$$= 3 \left[x(x-4) - y(x-4) \right]$$

$$= 3(x-4)(x-y)$$

6. $2x^2y - 2xy - 2xy^2 + 2y^2$

$$= 2y (x^2 - x - xy + y)$$

$$= 2y (x(x-1) - y(x-1))$$

$$= 2y (x-1)(x-y)$$

$$7. \quad -4x^2 + 8xy - 20x + 40y$$

$$8. \quad 60x^2 - 30xy + 10xy^2 - 5y^3$$

$$7. \quad -4x^2 + 8xy - 20x + 40y$$

$$= -4 \left[\underbrace{x^2 - 2xy} + \underbrace{5x - 10y} \right]$$

$$= -4 \left[x(x - 2y) + 5(x - 2y) \right]$$

$$= -4(x - 2y)(x + 5)$$

$$8. \quad 60x^2 - 30xy + 10xy^2 - 5y^3$$

$$= 5(12x^2 - 6xy + 2xy^2 - y^3)$$

$$= 5(6x(2x - y) + y^2(2x - y))$$

$$= 5(2x - y)(6x + y^2)$$

Trinomial Factoring

$$ax^2 + bx + c$$

*

$$\begin{aligned} \text{sum} &= b \\ \text{product} &= a \cdot c \end{aligned}$$

Trinomial Factoring

Each trinomial below can be factored with the short cut method (because $a = 1$). Factor each by using this method.

1. $x^2 + 8x + 15$

3. $x^2 + 6x - 16$

2. $n^2 - 8n - 9$

4. $m^2 - 6m + 9$

2. $n^2 - 8n - 9$
 $(n + 1)(n - 9)$

$a = 1$
 $b = -8$
 $c = -9$
Sum = -8
Product = -9
1, -9

1. $x^2 + 8x + 15$

$= (x + 3)(x + 5)$

$a = 1$
 $b = 8$
 $c = 15$

+ Sum = 8
• Product = 15
3, 5

Check

$(x + 3)(x + 5)$

$= x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$

3. $x^2 + 6x - 16$

4. $m^2 - 6m + 9$

3. $x^2 + 6x - 16$
 $= (x - 2)(x + 8)$

$a = 1$
 $b = 6$
 $c = -16$

Sum = 6
Product = -16

-2, 8

4. $m^2 - 6m + 9$

$= (m - 3)(m - 3)$

$= (m - 3)^2$

Sum = -6

product = 9

-3, -3

3. $x^2 + 6x - 16$

4. $m^2 - 6m + 9$

3. $x^2 + 6x - 16$
 $= (x - 2)(x + 8)$

$a = 1$
 $b = 6$
 $c = -16$

Sum = 6
Product = -16

-2, 8

4. $m^2 - 6m + 9$

$= (m - 3)(m - 3)$

$= (m - 3)^2$

Sum = -6

product = 9

-3, -3

Factor each trinomial by using the decomposition method.

5. $2x^2 + 5x + 3$

7. $3x^2 + 8x + 4$

6. $6x^2 + 17x - 3$

8. $6x^2 - 5x - 1$

5. $2x^2 + 5x + 3$

sum = 5
product = 6

2, 3

$= \underline{2x^2 + 2x} + \underline{3x + 3}$

$= 2x(x+1) + 3(x+1)$

$= (x+1)(2x+3)$

using
grouping
here

6. $6x^2 + 17x - 3$

$= 6x^2 - x + 18x - 3$

$= x(6x-1) + 3(6x-1)$

$= (6x-1)(x+3)$

sum = 17
product = -18
-1, 18

7. $3x^2 + 8x + 4$

8. $6x^2 - 5x - 1$

7. $3x^2 + 8x + 4$

$= 3x^2 + 6x + 2x + 4$

$= 3x(x+2) + 2(x+2)$

$= (x+2)(3x+2)$

Sum = 8
product = 12
6, 2

8. $6x^2 - 5x - 1$

$= 6x^2 - 6x + x - 1$

$= 6x(x-1) + 1(x-1)$

$= (x-1)(6x+1)$

Sum = -5
product = -6
-6, 1

Factor each two-variable trinomial by using the decomposition method.

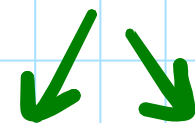
9. $2n^2 - 7mn - 15m^2$

11. $8a^2 - 14ab + 3b^2$

10. $9a^2 + 12ab + 4b^2$

12. $15p^2 - pq - 2q^2$

9. $2n^2 - 7mn - 15m^2$



$S = -7$
 $P = -30$

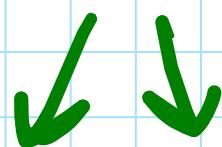
$3, -10$

$= \underbrace{2n^2 + 3mn} - \underbrace{10mn - 15m^2}$

$= n(2n + 3m) - 5m(2n + 3m)$

$= \boxed{(2m + 3m)(n - 5m)}$

10. $9a^2 + 12ab + 4b^2$



$= 9a^2 + 6ab + 6ab + 4b^2$

$S = 12$

$P = 36$

$6, 6$

$= 3a(3a + 2b) + 2b(3a + 2b)$

$= (3a + 2b)(3a + 2b) = \boxed{(3a + 2b)^2}$

11. $8a^2 - 14ab + 3b^2$

12. $15p^2 - pq - 2q^2$

11. $8a^2 - 14ab + 3b^2$

$\downarrow \downarrow$
 $= 8a^2 - 2ab - 12ab + 3b^2$

$= 2a(4a - b) - 3b(4a - b)$

$= \boxed{(4a - b)(2a - 3b)} \checkmark$

$S = -14$
 $P = 24$

$-2, -12$

12. $15p^2 - pq - 2q^2$

$\downarrow \downarrow$
 $= 15p^2 + 5pq - 6pq - 2q^2$

$= 5p(3p + q) - 2q(3p + q) = \boxed{(3p + q)(5p - 2q)}$

$S = -1$
 $P = -30$

$5, -6$

Factor each trinomial. Remember to first look for a common factor.

13. $6x^3 + 33x^2 + 45x$

15. $20b^2 + 70b + 60$

14. $24x^2 - 72xy + 54y^2$

16. $4x^2y - 17xy - 15y$

$$\begin{aligned} 14. \quad & 24x^2 - 72xy + 54y^2 \\ & = 6(4x^2 - 12xy + 9y^2) \\ & = 6(2x - 3y)^2 \end{aligned}$$

factor
this
trinomial

$$S = 11$$

$$P = 30$$

$$5, 6$$

13. $6x^3 + 33x^2 + 45x$

$$= 3x(2x^2 + 11x + 15)$$

$$= 3x[2x^2 + 5x + 6x + 15]$$

$$= 3x[x(2x+5) + 3(2x+5)]$$

$$= 3x(2x+5)(x+3)$$

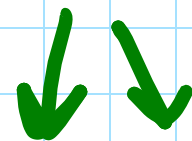
15. $20b^2 + 70b + 60$

16. $4x^2y - 17xy - 15y$

15.

$$20b^2 + 70b + 60$$

$$= 10(2b^2 + 7b + 6)$$



$$= 10(2b^2 + 3b + 4b + 6)$$

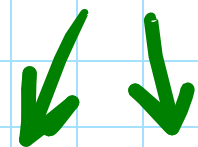
$$= 10(b(2b+3) + 2(2b+3))$$

$$= 10(2b+3)(b+2)$$

$$\begin{aligned} S &= 7 \\ P &= 12 \\ &3, 4 \end{aligned}$$

16. $4x^2y - 17xy - 15y$

$$= y(4x^2 - 17x - 15)$$



$$= y[4x^2 - 20x + 3x - 15]$$

$$= y[4x(x-5) + 3(x-5)]$$

$$\begin{aligned} S &= -17 \\ P &= -60 \end{aligned}$$

$$-20, 3$$

$$= y(x-5)(4x+3)$$

Difference of Squares

$$A^2 - B^2 = (A - B)(A + B)$$

Check

$$(A - B)(A + B) = A^2 + \cancel{AB} - \cancel{AB} - B^2 = A^2 - B^2$$

Factor each expression using difference of squares.

1. $4x^2 - 1$

3. $1 - 49x^2$

2. $64d^2 - 25$

4. $16t^2 - 121$

1. $4x^2 - 1 = (2x - 1)(2x + 1)$

2. $64d^2 - 25 = (8d - 5)(8d + 5)$

3. $1 - 49x^2 = (1 - 7x)(1 + 7x)$

4. $16t^2 - 121 = (4t - 11)(4t + 11)$

Factor each two-variable binomial by using difference of squares.

5. $25x^2 - 36y^2$

7. $x^2y^2 - 4$

6. $100u^2 - 81v^2$

8. $n^4 - 36m^2$

5. $25x^2 - 36y^2 = (5x - 6y)(5x + 6y)$

6. $100u^2 - 81v^2 = (10u - 9v)(10u + 9v)$

7. $x^2y^2 - 4 = (xy - 2)(xy + 2)$

8. $n^4 - 36m^2 = (n^2 - 6m)(n^2 + 6m)$

Factor each binomial. Remember to first look for a common factor.

9. $50y^2 - 72$

11. $36c - 121cd^2$

10. $x^2z - 25y^2z$

12. $20a^2b - 5bc^2$

10. $x^2z - 25y^2z$
 $= z(x^2 - 25y^2)$
 $= z(x + 5y)(x - 5y)$

12. $20a^2b - 5bc^2$
 $= 5b(4a^2 - c^2) = 5b(2a - c)(2a + c)$

9. $50y^2 - 72$
 $= 2(25y^2 - 36)$
 $= 2(5y + 6)(5y - 6)$

11. $36c - 121cd^2$
 $= c(36 - 121d^2)$
 $= c(6 + 11d)(6 - 11d)$

