Factoring Polynomials

any # terms 1. Common Factoring ← * Always try this first!

4 terms 2. Grouping Factoring

3 terms 3. Trinomial Factoring "$a=1$" (Easy)

"$a \neq 1$" (Harder) Decomposition Method

2 terms 4. Difference of Squares

- Factoring means to write as a product

- Factor 15

$$15 = (3)(5)$$

Simplify ("Multiply it out")

$$2x(5x+3) = 10x^2 + 6x$$

Factoring is the opposite of this

Factor the following binomials by using common factoring.

1. $4x - 10$
2. $x^5 - 2x^3$
3. $3x^2 - 12x$
4. $5x^3 + 20x^2$
5. $2x^2y - 16xy^2$
6. $-x^3 + x^4y$

1. $4x-10 = 2(2x-5)$

2. $x^5-2x^3 = x^3(x^2-2)$

3. $3x^2-12x = 3x(x-4)$

4. $5x^3+20x^2 = 5x^2(x+4)$

5. $2x^2y-16xy^2$

$=2xy(x-8y)$

6. $-x^3+x^4y$

$=-x^3(1-xy)$

Factor the following trinomials by using common factoring.

7. $6m^3n^2 - 2m^2n^3 + 4m^2n^2$

8. $9xyz - 3x^2y + 6xyz^2$

9. $16p^2q^2 - 8p^2q^4 - 24p^4q^2$

7. $6m^3n^2 - 2m^2n^3 + 4m^2n^2$

$= 2m^2n^2(3m - n + 2)$

8. $9xyz - 3x^2y + 6xyz^2$

$= 3xy(3z - x + 2z^2)$

9. $16p^2q^2 - 8p^2q^4 - 24p^4q^2$

$= 8p^2q^2(2 - q^2 - 3p^2)$

Factor each expression below by identifying a binomial that is a common factor.

10. $3x(x+y)-4(x+y)$

11. $2a(a-b)+5b(a-b)$

12. $3p(p+5)-7(p+5)$

10. $3x(x+y)-4(x+y) = (x+y)(3x-4)$

11. $2a(a-b)+5b(a-b) = (a-b)(2a+5b)$

12. $3p(p+5)-7(p+5) = (p+5)(3p-7)$

Factor each expression by using grouping.

1. $ay + a + 4y + 4$
2. $x^3 + 2x^2 + 3x + 6$
3. $x^3 - 7x^2y - 3xy + 21y^2$
4. $xy + 3xz + 2yz + 6z^2$

1. $ay + a + 4y + 4 = a(y+1) + 4(y+1)$

$= \boxed{(y+1)(a+4)}$

2. $x^3 + 2x^2 + 3x + 6 = x^2(x+2) + 3(x+2) = \boxed{(x+2)(x^2+3)}$

3. $x^3 - 7x^2y - 3xy + 21y^2 = x^2(x-7y) - 3y(x-7y)$

$= \boxed{(x-7y)(x^2-3y)}$

4. $xy + 3xz + 2yz + 6z^2 = x(y+3z) + 2z(y+3z)$

$= \boxed{(y+3z)(x+2z)}$

Factor each expression by using grouping. Remember to look for a common factor first.

5. $3x^2 - 12x - 3xy + 12y$

6. $2x^2y - 2xy - 2xy^2 + 2y^2$

7. $-4x^2 + 8xy - 20x + 40y$

8. $60x^2 - 30xy + 10xy^2 - 5y^3$

5. $3x^2-12x-3xy+12y$

$= 3[x^2-4x-xy+4y]$

$= 3[x(x-4)-y(x-4)]$

$= 3(x-4)(x-y)$

6. $2x^2y-2xy-2xy^2+2y^2$

$= 2y(x^2-x-xy+y)$

$= 2y(x(x-1)-y(x-1))$

$= 2y(x-1)(x-y)$

7. $-4x^2 + 8xy - 20x + 40y$

8. $60x^2 - 30xy + 10xy^2 - 5y^3$

7. $-4x^2+8xy-20x+40y$

$= -4[x^2-2xy+5x-10y]$

$= -4[x(x-2y)+5(x-2y)]$

$= -4(x-2y)(x+5)$

8. $60x^2-30xy+10xy^2-5y^3$

$=5(12x^2-6xy+2xy^2-y^3)$

$=5(6x(2x-y)+y^2(2x-y))$

$=5(2x-y)(6x+y^2)$

Trinomial Factoring

$$ax^2 + bx + c$$

* SUM = b
  product = a · c

Trinomial Factoring

Each trinomial below can be factored with the short cut method (because $a = 1$). Factor each by using this method.

1. $x^2 + 8x + 15$
2. $n^2 - 8n - 9$
3. $x^2 + 6x - 16$
4. $m^2 - 6m + 9$

1. $x^2+8x+15$

$=(x+3)(x+5)$

$a=1$
$b=8$
$c=15$

+ sum = 8
· product = 15

3, 5

Check

$(x+3)(x+5)$

$= x^2+5x+3x+15$

$= x^2+8x+15$

2. n^2-8n-9

$(n+1)(n-9)$

$a=1$
$b=-8$
$c=-9$

sum = -8
product = -9

1, -9

3. $x^2 + 6x - 16$

4. $m^2 - 6m + 9$

3. $x^2+6x-16$

$=(x-2)(x+8)$

$a=1$

$b=6$

$c=-16$

Sum $=6$

Product $=-16$

$-2, 8$

4. m^2-6m+9

$=(m-3)(m-3)$

$=(m-3)^2$

Sum $=-6$

product $=9$

$-3, -3$

3. $x^2 + 6x - 16$

4. $m^2 - 6m + 9$

3. $x^2+6x-16$

$=(x-2)(x+8)$

$a=1$

$b=6$

$c=-16$

Sum $=6$

Product $=-16$

$-2, 8$

4. m^2-6m+9

$=(m-3)(m-3)$

$=(m-3)^2$

Sum $=-6$

product $=9$

$-3, -3$

Factor each trinomial by using the decomposition method.

5. $2x^2 + 5x + 3$

7. $3x^2 + 8x + 4$

6. $6x^2 + 17x - 3$

8. $6x^2 - 5x - 1$

5. $2x^2+5x+3$

sum = 5
product = 6
2, 3

$= 2x^2+2x+3x+3$

$= 2x(x+1)+3(x+1)$

$= (x+1)(2x+3)$

Using grouping here

6. $6x^2+17x-3$

sum = 17
product = −18
−1, 18

$= 6x^2-x+18x-3$

$= x(6x-1)+3(6x-1)$

$= (6x-1)(x+3)$

7. $3x^2 + 8x + 4$

8. $6x^2 - 5x - 1$

7. $3x^2+8x+4$

Sum = 8

product = 12

6, 2

$= 3x^2+6x+2x+4$

$= 3x(x+2)+2(x+2)$

$= (x+2)(3x+2)$

8. $6x^2-5x-1$

Sum = -5

product = -6

-6, 1

$= 6x^2-6x+x-1$

$= 6x(x-1)+1(x-1)$

$= (x-1)(6x+1)$

Factor each two-variable trinomial by using the decomposition method.

9. $2n^2 - 7mn - 15m^2$

10. $9a^2 + 12ab + 4b^2$

11. $8a^2 - 14ab + 3b^2$

12. $15p^2 - pq - 2q^2$

9. $2n^2 - 7mn - 15m^2$

$S = -7$
$P = -30$
$3, -10$

$= \underbrace{2n^2 + 3mn - 10mn - 15m^2}$

$= n(2n+3m) - 5m(2n+3m)$

$= \boxed{(2n+3m)(n-5m)}$

10. $9a^2 + 12ab + 4b^2$

$S = 12$
$P = 36$
$6, 6$

$= 9a^2 + 6ab + 6ab + 4b^2$

$= 3a(3a+2b) + 2b(3a+2b)$

$= (3a+2b)(3a+2b) = \boxed{(3a+2b)^2}$

11. $8a^2 - 14ab + 3b^2$

12. $15p^2 - pq - 2q^2$

11. $8a^2 - 14ab + 3b^2$

$S = -14$

$P = 24$

$-2, -12$

$= 8a^2 - 2ab - 12ab + 3b^2$

$= 2a(4a-b) - 3b(4a-b)$

$= \boxed{(4a-b)(2a-3b)}$ ✓

12. $15p^2 - pq - 2q^2$

$S = -1$

$P = -30$

$5, -6$

$= 15p^2 + 5pq - 6pq - 2q^2$

$= 5p(3p+q) - 2q(3p+q) = \boxed{(3p+q)(5p-2q)}$

Factor each trinomial. Remember to first look for a common factor.

13. $6x^3 + 33x^2 + 45x$

14. $24x^2 - 72xy + 54y^2$

15. $20b^2 + 70b + 60$

16. $4x^2y - 17xy - 15y$

13. $6x^3+33x^2+45x$

$=3x(2x^2+11x+15)$

factor this trinomial

$S=11$

$P=30$

$5,6$

$=3x[2x^2+5x+6x+15]$

$=3x[x(2x+5)+3(2x+5)]$

$=3x(2x+5)(x+3)$

14. $24x^2-72xy+54y^2$

$=6(4x^2-12xy+9y^2)$

$=6(2x-3y)^2$

15. $20b^2 + 70b + 60$

16. $4x^2y - 17xy - 15y$

15. $20b^2+70b+60$

$= 10(2b^2+7b+6)$

$S=7$
$P=12$
$3,4$

$= 10(2b^2+3b+4b+6)$

$= 10(b(2b+3)+2(2b+3))$

$= 10(2b+3)(b+2)$

16. $4x^2y-17xy-15y$

$= y(4x^2-17x-15)$

$S=-17$
$P=-60$
$-20, 3$

$= y[4x^2-20x+3x-15]$

$= y[4x(x-5)+3(x-5)]$

$= y(x-5)(4x+3)$

Difference of Squares

$$A^2 - B^2 = (A - B)(A + B)$$

Check $(A-B)(A+B) = A^2 + AB - AB - B^2 = A^2 - B^2$

Factor each expression using difference of squares.

1. $4x^2 - 1$
2. $64d^2 - 25$
3. $1 - 49x^2$
4. $16t^2 - 121$

1. $4x^2-1 = (2x-1)(2x+1)$

2. $64d^2-25 = (8d-5)(8d+5)$

3. $1-49x^2 = (1-7x)(1+7x)$

4. $16t^2-121 = (4t-11)(4t+11)$

Factor each two-variable binomial by using difference of squares.

5. $25x^2 - 36y^2$

6. $100u^2 - 81v^2$

7. $x^2y^2 - 4$

8. $n^4 - 36m^2$

5. $25x^2 - 36y^2 = (5x - 6y)(5x + 6y)$

6. $100u^2 - 81v^2 = (10u - 9v)(10u + 9v)$

7. $x^2y^2 - 4 = (xy - 2)(xy + 2)$

8. $n^4 - 36m^2 = (n^2 - 6m)(n^2 + 6m)$

Factor each binomial. Remember to first look for a common factor.

9. $50y^2 - 72$

10. $x^2z - 25y^2z$

11. $36c - 121cd^2$

12. $20a^2b - 5bc^2$

9. $50y^2 - 72$

$= 2(25y^2 - 36)$

$= \boxed{2(5y+6)(5y-6)}$

10. $x^2z - 25y^2z$

$= z(x^2 - 25y^2)$

$= \boxed{z(x+5y)(x-5y)}$

11. $36c - 121cd^2$

$= c(36 - 121d^2)$

$= \boxed{c(6+11d)(6-11d)}$

12. $20a^2b - 5bc^2$

$= 5b(4a^2 - c^2) = \boxed{5b(2a-c)(2a+c)}$